

INFLUENCE OF INITIAL DEFLECTION ON BEARING CAPACITY FOR MICRO PILES

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ABSTRACT

In Scandinavian countries micro piles usually means slender, point bearing steel tube piles surrounded by soft clay. For such piles buckling comes into play. Methods for calculation of axial load capacity for such piles have been developed since 1926. In this paper the present used methods for evaluation of the load capacity for micro piles in soft soils are reviewed. The analysis takes into account the effect of initial horizontal deflection of the pile and the elastic-plastic response of the soil as well as the influence of load duration and built in stresses in the pile material.

INTRODUCTION

When a foundation pile is surrounded by a very soft soil, it is obvious that the pile is similar to an ordinary above ground situated column. Thus the pile must therefore be checked for buckling. Such soft soils and long, small diameter piles are common in the Scandinavian countries. Therefore the issues concerning buckling for piles have been given attention there since long time ago. In many other countries, where very soft soils are uncommon, or not even exist, pile designers are often surprised when they hear about buckling as a possible failure mode for foundation piles.

Forsell (1927) calculated the buckling load for a strut surrounded by a elastic media with constant properties. Bjerrum (1956) used his results to calculate parameters at a number of load tests where slender steel piles were tested to failure. Broms (1965) presented a method to calculate the critical load for a pile with a particular initial curvature. His work was expanded by Bernander and Svensk (1970) to include a bilinear elastic-plastic response of the soil. The Swedish Pile Commission has presented methods for the calculation of pile load capacity taking buckling and the additional factors mentioned above into account (ref 4,5,7, 9).

These methods are today used as standard tools by pile designers not only for micro piles, but also for precast 300*300 mm driven concrete piles. The calculation methods presented have made it possible to assign very large loads to small diameter steel tube piles and steel core piles.

CALCULATION METHODS

The dominant parameters involved in a buckling analysis for a foundation pile surrounded by soil are listed below. They are:

- the pile length
- geometry of the pile
- material properties of the pile
- material properties of the soil
- boundary conditions for pile tip and pile top

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Each of these mentioned basic properties can be split further:

pile geometry :

- changes in cross section
- properties of connections between different pile segments
- initial deflection and curvature (3-D)

pile material properties :

- elastic - plastic properties
- interaction between different materials (usually steel and concrete)
- creep properties

soil properties

- variation of parameters along the pile
- stress - strain relationship for horizontal loading
- influence of load duration

pile boundary conditions

- usually a ball point connection is assumed at top and tip, but pile tips cast into bedrock are also common, resulting in fixed conditions there.
- the direction of the external load(s) during the load-displacement sequence
- in case of a pile cross section with different materials, the distribution of the load on different part of the section results in different pile capacity

ADVANCED MODELS FOR CALCULATION

A model which take all the parameters mentioned above into consideration will of course be rather complicated, although possible using advanced structural software, as for example ABACUS. Such tools have in common that they require advanced knowledge of their use. Such skills are not in the hands of a foundation designer. Therefore, simplified models which can be handled without knowledge of advanced structural software have been created.

Considering the rate of improvement in computer software, it can be expected that calculation tools that are easy to use, but employing with very sophisticated models hidden for the user, will show up.

SIMPLIFIED MODEL

In Scandinavia, a model presented by the Swedish Pile Commission has been used in foundation engineering practice for a number of years. The features of the model are as follows:

1. infinitely long pile completely surrounded by a bilinear elastic-plastic media
2. hinged connection at top and tip
3. constant soil and pile properties along the pile length
4. initial sine-formed deflection in 2D
5. no stresses in pile and soil due to initial deflection before loading the pile top
6. built in stresses are taken into account as an assumed initial deflection and a reduction in the E-modulus.
7. bi-linear elastic-plastic horizontal soil response

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The assumptions in the list above are of course rarely fulfilled for any real pile. As for example, a pile must have a finite length, the deflection takes place in 3D, there will be some initial stresses before loading, etc. Nevertheless, this model is routinely used to design slender steel tube piles to very high capacities. The model is built in most building codes handling foundation piles.

EVALUATION OF EXPRESSIONS FOR PILE CAPACITY

The basic differential equation for a axially loaded beam in an elastic media was given by Forsell (ref 6) :

$$EIy^{IV} + Fy'' + kdy = 0 \quad \dots (1)$$

where

$$y^n = d^n y / dx^n$$

x = length coordinate
= deflection

F=axial force in the beam

EI = flexural stiffness (E=Young modulus, I=moment of inertia)

ykd = contact reaction between the beam and the soil that cause a deflection y =1

d= diameter of the pile

In solving eq (1), the direction of the load applied is assumed constant (usually vertical) during the load-deflection evaluation, a so called gyroscopic system.

The value of the elastic media (the soil) modulus k (also called coefficient of subgrade reaction and similar), is in defined by the expression

$$q = k \cdot y \quad \dots (2)$$

where q= contact pressure between pile and soil

Therefore, the dimension of k is Force/Length³. The value kd is the pressure required to give the pile the deflection y=1. The dimension of kd is Force/Length². If the soil is assumed to compress only a distance = d from the pile surface, a common engineering assumption since the load increase for larger distances is pretty small, the strain ϵ and the deflection y for a given pressure q can be calculated as

$$\epsilon = y/d = q/E_{soil}, \text{ which gives} \quad \dots (3)$$

$$y = qd / E_{soil} \quad \dots (4)$$

The value kd per definition gives y = 1, thus from eq (4) :

$$kd = E_{soil} \quad \dots (5)$$

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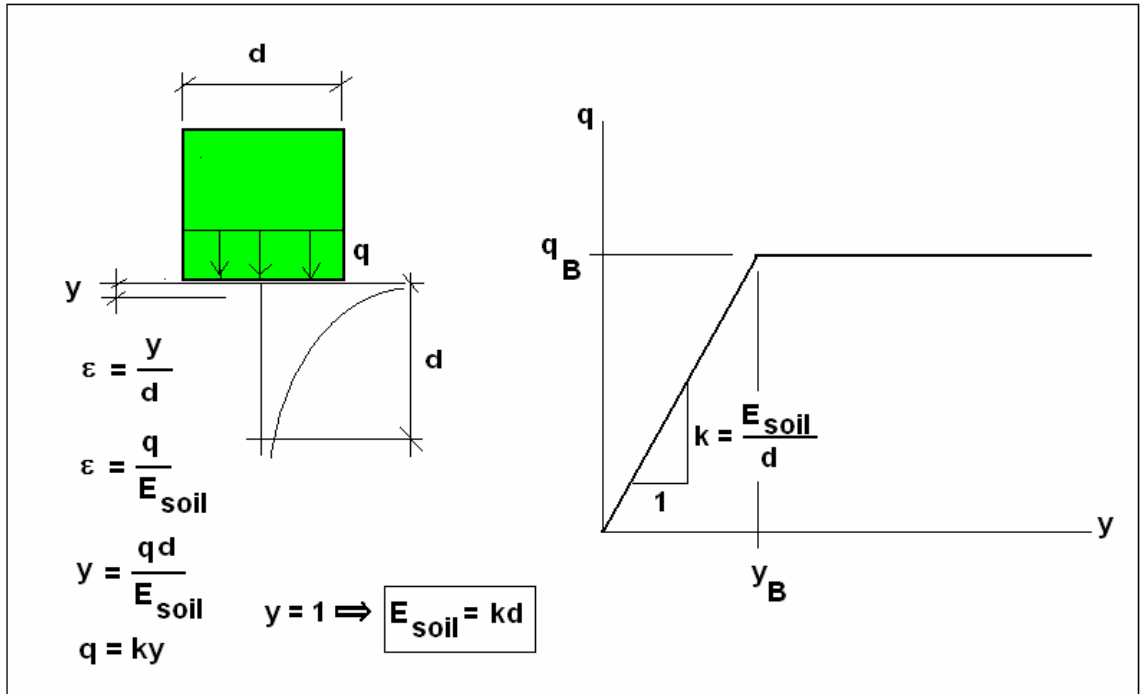


Figure 1. Definitions for deflection and contact pressure between pile and soil.

For cohesive soils E_{soil} is usually assumed to be a function $M(t)$ of the undrained shear strength c_u and the time the pressure q acts (t is time) :

$$E_{soil} = M c_u \quad \dots(6)$$

where

c_u = undrained shear strength

$M = 50$ for long time loading

$M = 200$ for short time loading.

The soil is assumed to behave bilinear elastic-plastic, se figure 1. The yield pressure q_B is assumed to be a multiple of the undrained shear strength for cohesive soils,

$$q_B = N \cdot c_u \quad \dots (7)$$

where $N = 6$ for long time loading

$N = 9$ for short time loading

The value of the yield deflection y_B (figure 1) varies between

$$y_B = (6/50)d = 0.12d \quad \text{for long time loading} \quad \dots(8)$$

and

$$y_B = (9/200)d = 0.045d \quad \text{for short time loading} \quad \dots (9)$$

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“Short” and “long” can be calculated from a consolidation calculation in horizontal direction. In that way, the soil permeability, the pile diameter and the time of loading can be considered. Of course, the meaning of “short” and “long” is quite different for a large diameter pile and a small diameter micro pile. Nevertheless, it is often assumed that “long time loading” means a load acting a week, or longer.

A linear variation between $M=50$ and $M=200$ is assumed and a time-factor T is defined as

$T = 0$ for short time loading

$T = 1$ for long time loading

For intermediate values, $0 < T < 1$, E_{soil} and y_B can be calculated as

$$E_{soil} = (200/(1+3*T))*c_u \quad \dots(10)$$

and

$$y_B = ((9+24T-9T^2)/200)*d \quad \dots (11)$$

The value of T is often expressed as a percentage, 0 % - 100 %. For buildings T is usually around 90 %, for railway bridges around 50 % and for structures mainly loaded by wind even smaller.

For friction soils (sand, gravel) the modulus of subgrade reaction is usually assumed to increase with depth, but often a constant value is assumed for such soils, making it possible to use the same calculation methods for both cohesive and friction soils. The values vary typically between $E_{soil} = 5$ MPa for loose soils up to 30 MPa for dense soils. These values are assumed independent of time.

INITIAL STRAIGHT BEAM

Forsell (ref 6) solved the differential equation (1) (an axially loaded straight beam in an elastic media), assuming ballpoint connections at top and tip. The buckling load F_c for such a beam is

$$F_c = 2(kdEI)^{1/2} \quad \dots(12)$$

or

$$F_c = 2(E_{soil}EI)^{1/2} \quad \dots (13)$$

$$\text{using the relationship } kd = E_{soil} \quad \dots (5)$$

The length between the points of inflection, that is, the buckling length L_c , is then

$$L_c = \pi (EI/kd)^{1/4} \quad \dots(14)$$

or

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$$L_c = \pi(EI/E_{soil})^{1/4} \quad \dots(15)$$

INITIALLY DEFLECTED BEAM

The expression for F_c above is valid for an initially perfect straight beam. Bernander & Svensk (4) found that the relationship between deflection y and axial load F for an initially deflected pile can be expressed as a function of the increasing deflection y_{omax} :

$$F(y_{omax}) = F_c \cdot (1 / (1 + y_{imax} / y_{omax})), \quad y_{omax} > 0 \quad \dots (16)$$

when assuming an initial sine-formed deflection

$$y_o(x) = y_{imax} \cdot \sin(\pi x / L_c) \quad \dots(17)$$

and also a sine-formed additional deflection caused by the axial load applied

$$y_i(x) = y_{omax} \cdot \sin(\pi x / L_c) \quad \dots (18)$$

The deflection functions are illustrated in figure 2.

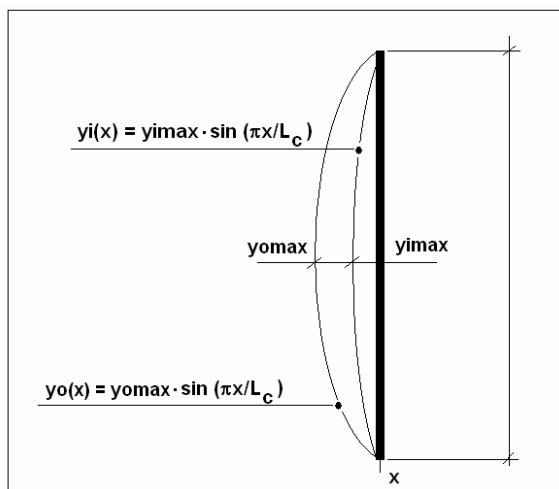


Figure 2. Assumed initial and additional deflections $y_i(x)$ and $y_o(x)$.

INFLUENCE OF ELASTIC-PLASTIC BEHAVIOUR OF THE SOIL

Eq (16) says that F approaches F_c as y_{omax} increases, provided that the soil is elastic. If instead a bi-linear elastic-plastic behaviour as illustrated in figure 1 is assumed, Eq 16 can still be used if the value of k_d is modified. The modified value is called k_{ed} .

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In figure 3 the situation over a length of L_c is illustrated. The contact pressure between the pile and the soil is elastic to some distance from the end points of the segment. At a part around the middle of the beam element the displacement has exceeded the yield deflection y_B causing a plastic reaction q_B in this region. The length of the plastic region is $2x_B$.

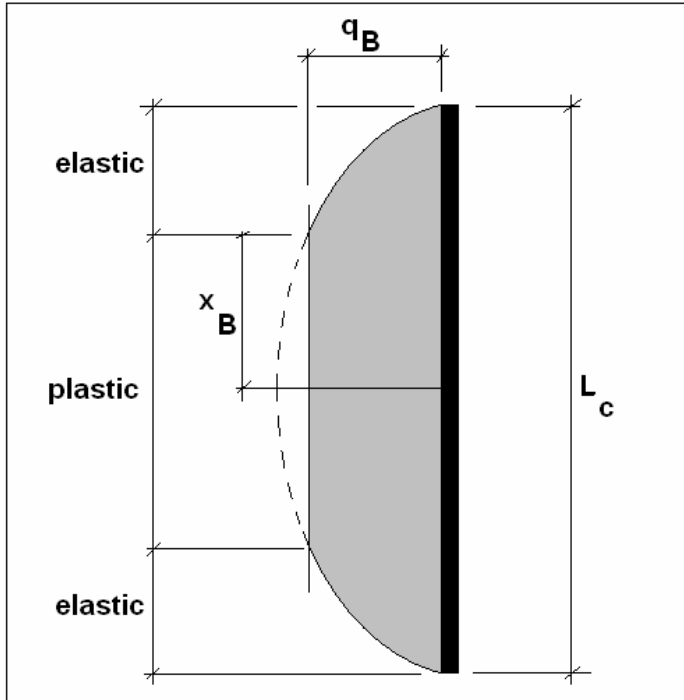


Figure 3. Soil yield over a part of the pile segment L_c .

The modified modulus k_{ed} is now chosen so that the sum of contact reaction over the element length L_c is the same as if the bi-linear pressure - deflection relationship illustrated in figure 2 should be used. Using the notations in figure 3, this assumption yields (ref 4)

$$k_{ed} = k_d \cdot f(y_{\max}) \quad \dots (19)$$

where the function $f(y_{\max})$ is given by the expression

$$f(y_{\max}) = q_B \cdot x_B \cdot \pi / (L_c \cdot y_{\max} \cdot k_d) + 1 - \sin(\pi \cdot x_B / L_c) \quad \dots (20)$$

The value of $f(y_{\max})$ is :

= 1 if $y_{\max} \leq y_B$ (no soil yielding).
 < 1 if $y_{\max} > y_B$ (soil yields)

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The expression for x_B (half length of the yield length) is given in ref (5).

Finally, combining eq (16), (19) and (20) gives

$$F(y_{\max}) = 2 \cdot (EI \cdot k_d \cdot f(y_{\max}))^{1/2} \cdot (1 / (1 + y_{\max} / y_{\max})) \quad \dots (21)$$

Using eq (21), the relationship between applied force F and additional deflection y_{\max} can be evaluated. The function yields a peak value for the applied load, as illustrated in figure 5 later in this paper.

Instead of assuming equal sum of reaction between pile and soil when the modified value k_{ed} is calculated, the 'principle of equivalent work' can be used. Solutions with this assumption are given in ref (4). This approach yields somewhat higher values of k_{ed} .

BENDING MOMENT CAUSED BY DEFLECTION

The relationship between bending moment M and additional deflection y_{\max} is evaluated:

$$M(y_{\max}) = Ely'' \quad \dots (22)$$

$$y'' = y_{\max} \cdot \pi^2 / L_c^2 \cdot \cos(\pi \cdot x / L_c) \quad \dots (23)$$

The peak bending moment corresponds to $x = L_c / 2$. Therefore

$$M_{\max}(y_{\max}) = EI \cdot \pi^2 \cdot y_{\max} / L_c^2 \quad \dots (24)$$

Combining eq (24) and eq (14), $L_c = \pi (EI / k_d)^{1/4}$, yields

$$M_{\max}(y_{\max}) = (EI \cdot k_d)^{1/2} \cdot y_{\max} \quad \dots (25)$$

Eq (16) can be written

$$(EI \cdot k_d)^{1/2} \cdot y_{\max} = F \cdot (y_{\max} + y_{\max}) / 2 \quad \dots (26)$$

Combining eq (25) and eq (26) finally yields

$$M_{\max}(y_{\max}) = F \cdot (y_{\max} + y_{\max}) / 2 \quad \dots (27)$$

PILE CAPACITY FROM MATERIAL STRENGTH

The ultimate capacity without considering buckling for the pile cross section loaded with an axial load and a bending moment can be evaluated from the expression

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$$F_{\text{capacity}}/F_{\text{load}} + M_{\text{capacity}}/M_{\text{load}} = 1 \quad \dots (28)$$

where

F_{capacity} and M_{capacity} are evaluated from the strength and dimensions of the pile section

F_{load} = applied axial load

M_{load} = corresponding bending moment, eq (27)

The value of the capacity of the cross section can be calculated using different assumptions. For example, on the conservative side is to assume that the capacity corresponds to reaching the stress yield for the outer surface of the section, that is

$$f_{\text{yield}} = F_{\text{load}}/A + M_{\text{load}}/W \quad \dots (29)$$

where A = cross section area

W = bending resistance modulus = I^2/d

Using eq (27) for the bending moment, the relationship between axial pile capacity can then be calculated:

$$F_{\text{capacity}} = f_a \cdot A / (1 + (y_{\text{omax}} + y_{\text{imax}}) \cdot A / (2 \cdot W)) \quad \dots (30)$$

Usually the capacity is calculated assuming some yielding for the pile material. As for example the value of W use to be multiplied by some factor, typically 1.10 to 1.25. The upper limit for the capacity for the pile section is evaluated assuming a complete plastic section where cross section equilibrium is obtained by yield stresses in all section elements.

EFFECTS OF BUILT IN STRESSES IN PILE SECTION

The built in stresses in a pile section made by steel can be considered by reducing the modulus of elasticity. Commonly a 10 % reduction is used,

$$E_{\text{red}} = E \cdot 0.9 \quad \dots (31)$$

Further, an assumed initial deflection y_{imax} is assumed. According to most building codes, the value depends on the type of section. For welded steel tube piles a value $0.0013L_c$ is often prescribed. For massive sections $0.0025L_c$ is used.

GEOMETRIC INITIAL DEFLECTION

The initial deflection caused by the curvature (projected in 2D) for the pile shall be evaluated over a length equal to L_c , see figure 4.

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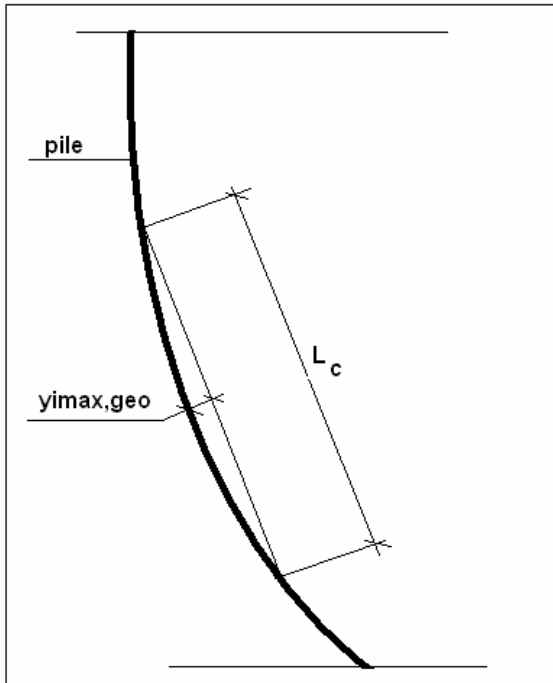


Figure 4 Initial geometric deflection.

Assuming that the curvature is a circle and using the 'chord-theorem' the initial geometric deflection, $y_{\text{imax,geo}}$, can be expressed

$$y_{\text{imax,geo}} = L_c^2 / (8 \cdot R) \quad \dots (32)$$

where R = measured radius of curvature of the pile

RESULTING INITIAL DEFLECTION

The assumed total initial deflection is obtained as the sum of the two above mentioned components, that is

$$y_{\text{omax}} = y_{\text{imax,a}} + y_{\text{imax,geo}} \quad \dots (33)$$

If the geometric deflection is not measured for the pile driven but instead assumed from empirical values of R , multiplication with a partial factor of safety, typically 1.5 to 2, is often used. Therefore it normally pays to measure the curvature if tube piles. The pile is usually assumed free of stresses in this state, as well as the surrounding soil.

EVALUATION OF PILE CAPACITY

When the different parameters have been evaluated, eq (21) and eq (30) are used to calculate the axial load F for different values of y_{omax} , see figure 5. The corresponding graph is plotted and the peak capacity of the pile is obtained.

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As can be seen from figure 5, two situations can occur:

Case 1. The peak buckling load is below the strength envelope.

Case 2. The peak buckling load extends above the strength envelope.

In the first case, the peak buckling load gives the capacity of the pile. In the second case, the intersection between the two curves gives the capacity. For steel tube micro piles in practice, the second case is the most common.

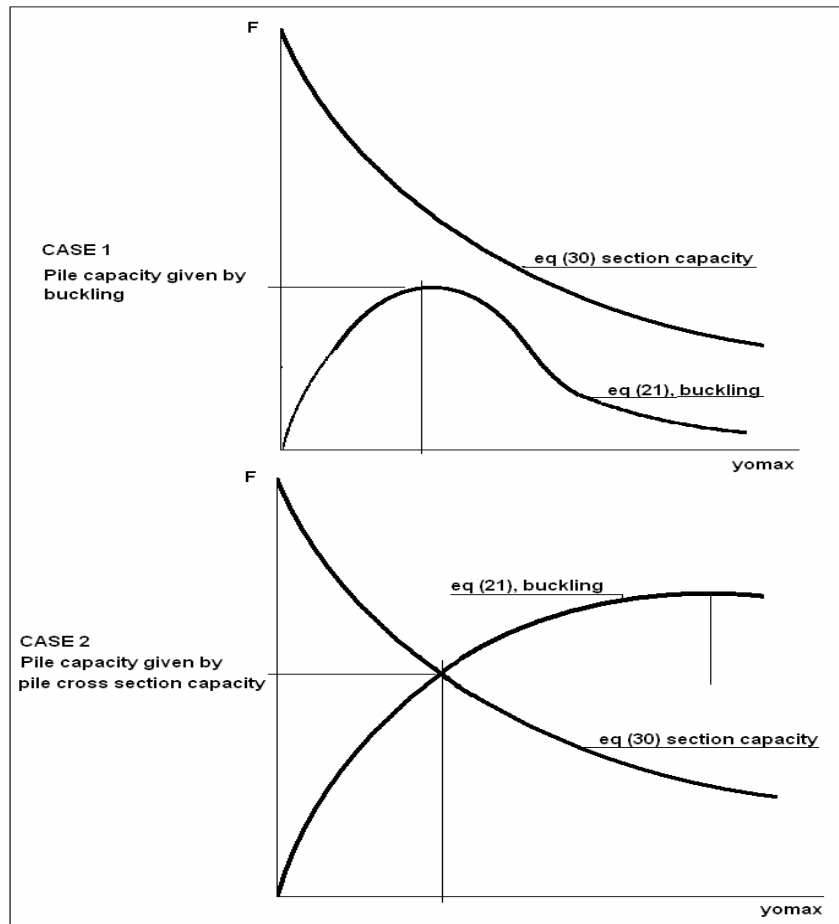


Figure 5. Evaluation of pile capacity from either one of two cases.

CALCULATION EFFORTS

Maybe needless to say, the calculations and presentations required are best made on PC. There are PC-programs available to carry out the numerical work and present the results on screen in order to make necessary modifications before a print out is made, ref (8).

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There are methods assuming hand calculations, but these are sometimes difficult to use, in particular for those making the calculations without previous experience or with limited knowledge of the theory behind the methods. It is recommended that the graphs shown in fig 5 are drawn, in order to decrease the probability for errors. For a PC, the graphs and the pile capacity shows up in an instant.

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